Exercise 33

For the following exercises, solve the equations over the complex numbers.

$$x^2 - 4x + 13 = 0$$

Solution

The two terms with x, x^2 and $4x$, cannot be combined, so it's necessary to complete the square to solve for x. Recall the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = -4$, which means $B = -2$ and $B^2 = 4$. Add and subtract 4 from the left side and apply the identity.

$$(x^2 - 4x + 4) + 13 - 4 = 0$$

$$(x + (-2))^2 + 9 = 0$$

$$(x - 2)^2 + 9 = 0$$

Now that x appears in only one place, it can be solved for. Subtract 9 from both sides.

$$(x - 2)^2 = -9$$

Take the square root of both sides.

$$\begin{aligned}\sqrt{(x-2)^2} &= \sqrt{-9} \\ &= \sqrt{9(-1)} \\ &= \sqrt{9}\sqrt{-1} \\ &= 3i\end{aligned}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x - 2$.

$$|x - 2| = 3i$$

Remove the absolute value sign by placing $\pm$ on the right side.

$$x - 2 = \pm 3i$$

Add 2 to both sides.

$$x = 2 \pm 3i$$

Therefore, $x = \{2 - 3i, 2 + 3i\}$.